Dobson's method (General Inverse FRF)

- In an SDoF system, we know that:

$$H(\omega)=\frac{1}{(k-m\omega^2)+jd}\Rightarrow H^{-1}(\omega)=(k-m\omega^2)+jd$$

- The rth mode of an FRF can be written as:

$$H(\omega)=\frac{{}_rA}{\lambda_r^2-\omega^2}+R$$

- where R is complex and related to contribution of modes to other than the one of current interest. (Error of considering SDoF instead MDoF). Consider:

$$\delta H(\omega)=H(\omega)-H(\Omega)=\frac{{}_rA}{\lambda_r^2-\omega^2}-\frac{{}_rA}{\lambda_r^2-\Omega^2}$$

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- And then:

$$\Delta(\omega) = \frac{\omega^2 - \Omega^2}{\delta H} = \frac{1}{{}_rA}\left(\lambda_r^2 - \omega^2\right)\left(\lambda_r^2 - \Omega^2\right) = \mathrm{Re}(\Delta) + j\,\mathrm{Im}(\Delta)$$

- It can be seen that Re(Δ) and Im(Δ), are related to the variable frequency (ω), so:

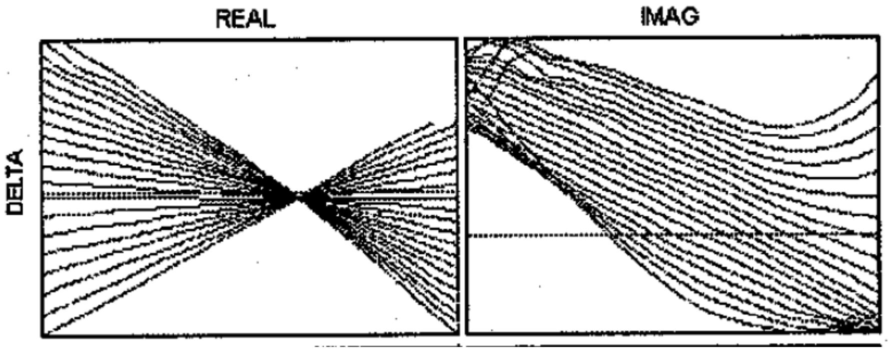
$$\mathrm{Re}(\Delta) = m_R\omega^2 + c_R \quad ; \quad \mathrm{Im}(\Delta) = m_I\omega^2 + c_I$$

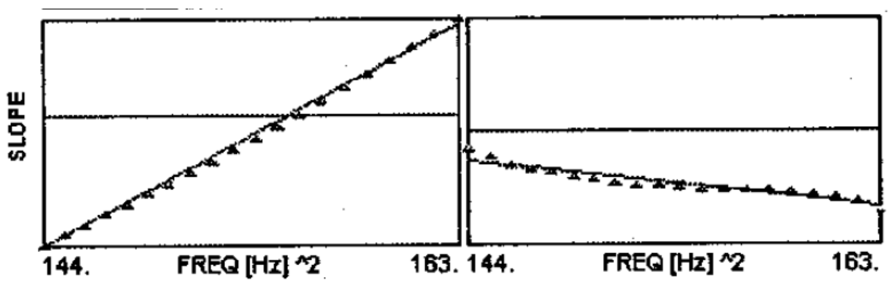
- where:

$$m_R = +a_r\left(\Omega^2 - \omega_r^2\right) - b_r\left(\eta_r\omega_r^2\right)$$
$$m_I = -b_r\left(\Omega^2 - \omega_r^2\right) - a_r\left(\eta_r\omega_r^2\right)$$

Procedure of Dobson's method

- Therefore, the first step of the Dobson's method can be made as follows:
  1. Using the FRF data measured in the vicinity of the resonance (ω_r) choose one of the measured points as the datum (fixed starting frequency Ω) and then calculate the possible values of $\Delta(\omega)$ Using the remaining data points.
  2. Plot Re and Im of $\Delta(\omega)$ vs. ω^2 and compute the best fit straight line and then determine m_R and m_I for the datum.





Procedure of Dobson's method

3. Plot graphs of $m_r(\Omega)$ and $m_I(\Omega)$ vs. Ω^2, using the results from the repeated application of step 1, each time using a different one of the measurement points as the fixed starting frequency (Ω)
4. Determine the slopes of the best-fit straight lines through these two plots, n_r and n_I and their intercepts with the vertical axis (d_R and d_I). In other word we have:

$$\begin{cases} m_R = n_R \Omega^2 + d_R \\ m_I = n_I \Omega^2 + d_I \end{cases}$$

Procedure of Dobson's method

5. Using these four quantities and the formula, determine the four modal parameters required for that mode. So we calculate η_r, ω_r and ${}_rA=a_R+jb_R$ ☺.

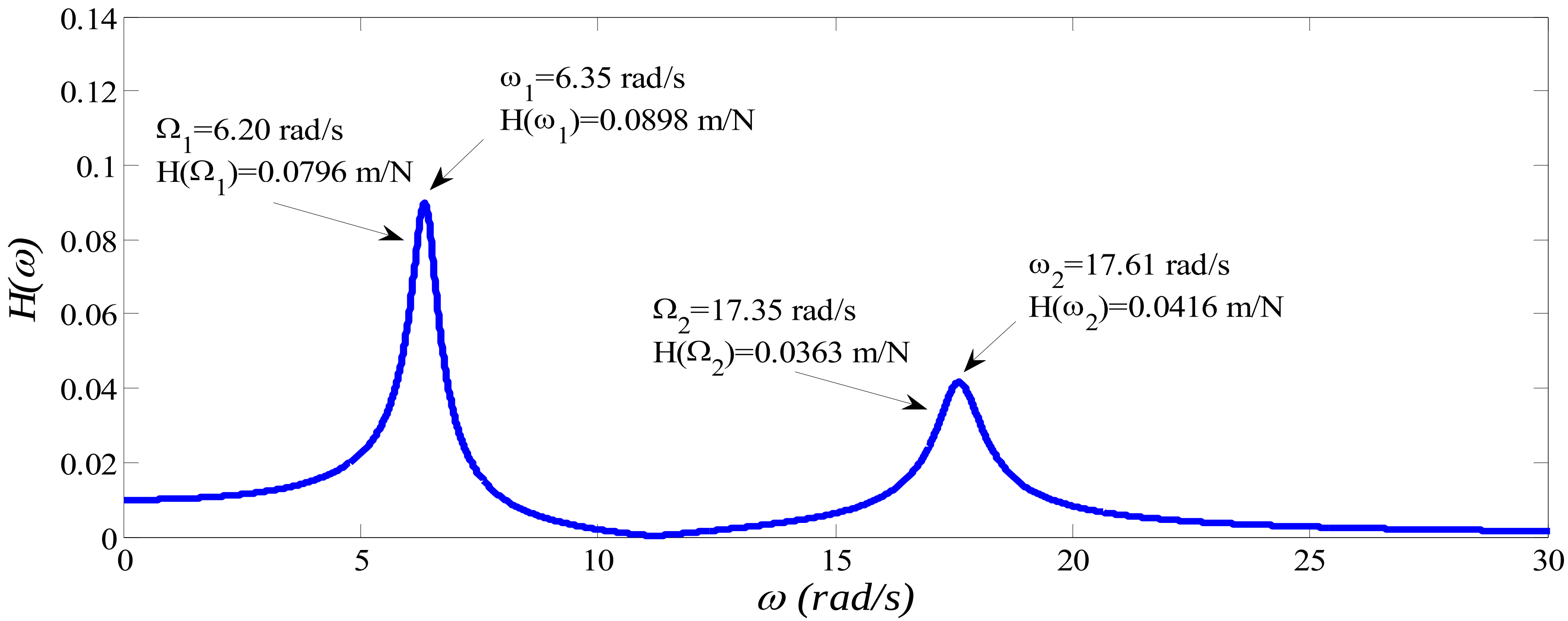
$$a_r = n_R \quad \& \quad b_r = -n_I$$

$$\begin{cases} d_R = -b_r\left(\eta_r\omega_r^2\right) - a_r\left(\omega_r^2\right) \\ d_I = -a_r\left(\eta_r\omega_r^2\right) + b_r\left(\omega_r^2\right) \end{cases} \Rightarrow \begin{cases} \omega_r \\ \eta_r \end{cases}$$

$$\begin{cases} a_R = -\dfrac{\omega_r^2}{d_R}\dfrac{1-\eta_r\left(n_I/n_R\right)}{1+\left(n_I/n_R\right)^2} \\ b_R = -a_R\left(\dfrac{n_I}{n_R}\right) \end{cases} \Rightarrow {}_rA = a_R + j\,b_R$$

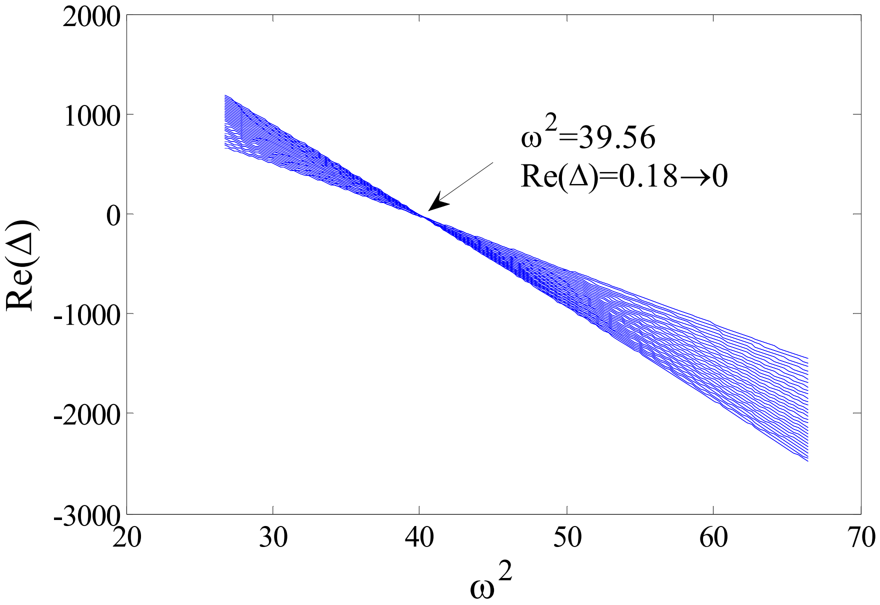
Example#1 (Dobson's Method)

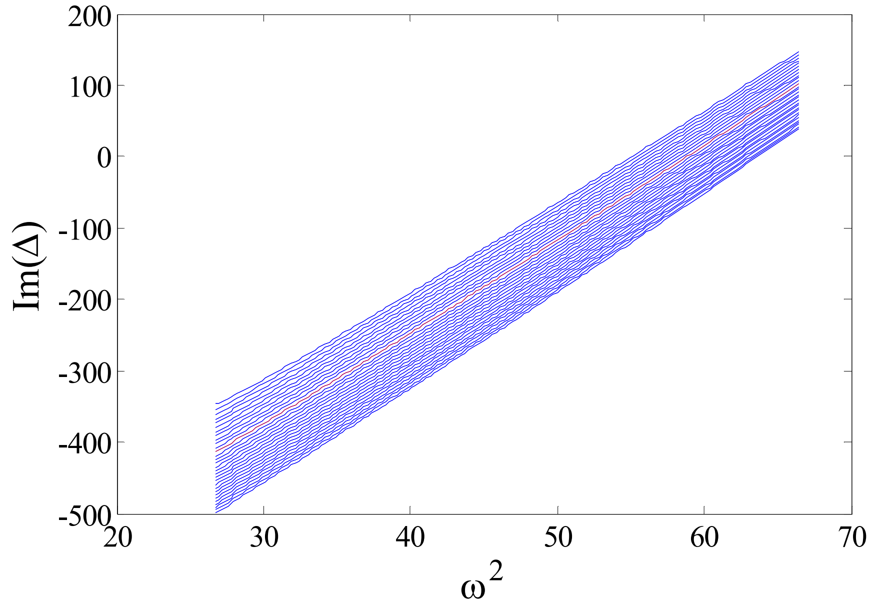
- **Example:** Find the modal parameters of in example#1 using the Dobson's method.



Example#1 (Dobson's Method)

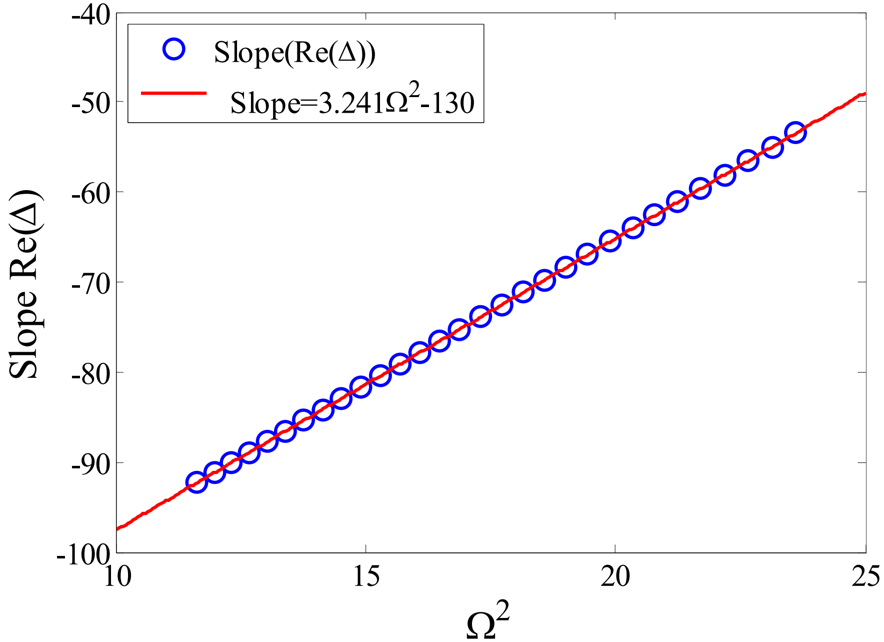
- Step#2: Plotting Re and Im of $\Delta(\omega)$ vs. ω^2

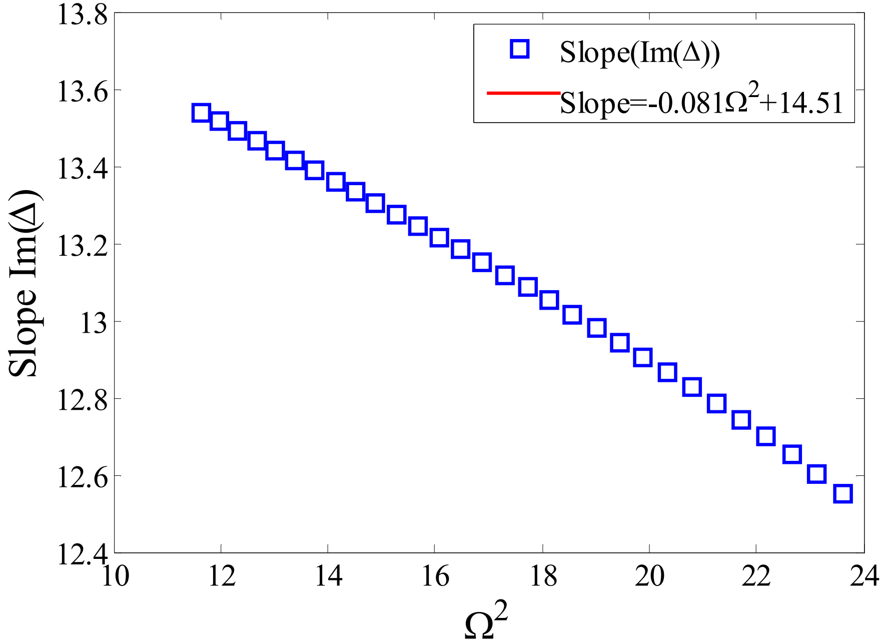




Example#1 (Dobson's Method)

- Then we have:





$$\begin{cases} m_R = n_R\Omega^2 + d_R \\ m_I = n_I\Omega^2 + d_I \end{cases} \Rightarrow \begin{cases} n_R = +3.241 \quad \& \quad d_R = -130 \\ n_I = -0.081 \quad \& \quad d_I = +14.51 \end{cases}$$

Example#1 (Dobson's Method)

- Then we have:

$$\begin{cases} d_R = -130 = -0.081\left(\eta_r\omega_r^2\right) - 3.241\left(\omega_r^2\right) \\ d_I = +14.5 = -3.241\left(\eta_r\omega_r^2\right) + 0.081\left(\omega_r^2\right) \end{cases} \Rightarrow \begin{cases} \eta_r\omega_r^2 = 3.469 \\ \omega_r^2 = 40.197 \end{cases}$$

$$\Rightarrow \begin{cases} \omega_r = 6.340\,\text{rad/sec} & \therefore \\ \eta_r = 0.0863 & \therefore \end{cases}$$

$$\begin{cases} a_R = \dfrac{40.197}{130}\dfrac{1-0.0863(0.025)}{1+(0.025)^2} = 0.308 \\ b_R = -0.308(0.025) = -0.008 \end{cases}$$

$$\Rightarrow {}_rA = 0.308 - 0.008j \quad \therefore$$

Example#1 (Comparison)

- Then we have:

$$\text{Peak-Picking method} \Rightarrow \begin{cases} \omega_1 = 6.35\,\text{rad/sec} \\ \eta_1 = 0.0866 \\ {}_1A = 0.315 \end{cases}$$

$$\text{Circle fit method} \Rightarrow \begin{cases} \omega_r = 6.35\,\text{rad/sec} \\ \eta_r = 0.0787 \\ {}_rA = 0.285 \end{cases}$$

$$\text{Dobson's method} \Rightarrow \begin{cases} \omega_r = 6.340\,\text{rad/sec} \\ \eta_r = 0.0863 \\ {}_rA = 0.308 - 0.008j \end{cases}$$